Bachelor of Science (B.Sc.) Semester—III (C.B.S.) Examination

STATISTICS

(Statistical Methods)

Paper—I

Time : Three Hours] [Maximum Marks : 50

N.B. :— All questions are compulsory and carry equal marks.

1. (A) Define (i) joint p.d.f. (ii) marginal p.d.f. (iii) conditional p.d.f. (iv) conditional mean and (v) conditional variance of a continuous bivariate probability distribution.

The p.d.f. of a continuous bivariate distribution is

$$f(x, y) = \begin{cases} x + y \quad , & 0 < x < 1 \\ & 0 < y < 1 \\ 0 \quad , & \text{elsewhere} \end{cases}$$

Find :

(i) Marginal p.d.f.s of X and Y.

(ii) Conditional p.d.f. of Y given X = x

(iii) Conditional mean of Y given $X = \frac{1}{2}$

(iv) Conditional variance of Y given $X = \frac{1}{2}$. 10

OR

(E) Define :

(i) Bivariate m.g.f.

(ii) Bivariate c.d.f.

(iii) Stochastic independence of two random variables.

If the r.v.s X and Y are independent, show that cov(X, Y) = 0. Is the converse true ? Justify.

A fair coin is tossed three times. Let X take a value 1 or 0 according as a head or a tail occurs on the first toss, and let Y denote the no. of heads which occur. Determine :

(i) the probability distributions of X and Y

(ii) the joint probability distribution of X and Y

(iii) cov(X, Y). 10

2. (A) State the p.d.f. of Bivariate normal distribution of r.v. (X, Y). Find its m.g.f. and hence find means of X and Y. Let X and Y have a bivariate normal distribution with means μ_1 and μ_2, positive variances σ_1^2 and σ_2^2 and correlation coefficient ρ. Then using m.g.f. show that X and Y are independent iff $\rho = 0$. 10

OR

(E) State the p.m.f. of multinomial distribution. Hence write p.m.f. of trinomial distribution. Find its m.g.f. Check whether the variables following trinomial distribution are independent.

A certain city has three television channels. During prime time on Saturday nights, channel 12 has 50% of the viewing audience, channel 10 has 30% of the viewing audience and channel 3 has 20% of the viewing audience. Find the probability that among eight television viewers in the city, randomly chosen on a Saturday night, two will be watching channel 12, three will be watching channel 10 and three will be watching channel 3. 10

3. (A) Let $X_1\ X_2\ ....\ X_n$ be a random sample of size n from exponential distribution. Find the probability distribution of $\sum_{i=1}^{n} X_i$. 10

(B) If the joint p.d.f. of random variables X_1 and X_2 is

$$f(x_1,\ x_2) = \begin{cases} e^{-(x_1+x_2)} & , \quad x_1 > 0,\ x_2 > 0 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Find :

(a) the joint p.d.f. of r.v.s $Y_1 = X_1 + X_2$ and $Y_2 = \dfrac{X_1}{X_1 + X_2}$

(b) the marginal p.d.f. of Y_2. 5+5

OR

(E) Let X be a geometric variable with probability distribution :

$$f(x) = \frac{3}{4}\left(\frac{1}{4}\right)^{x-1},\ x = 1,\ 2,\ 3,\ ......$$

Find the probability distribution of $Y = X^2$.

(F) If X is a standard normal variable, find the p.d.f. of $Y = X^{1/3}$.

(G) If Y = | X | show that $-\infty < x < +\infty, x \neq 0$

$$g(y) = \begin{cases} f(y) + f(-y) & , \quad y > 0 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

where f(x) is p.d.f. of X at x and g(y) is p.d.f. of Y at y.

(H) Define :

(i) Statistic and parameter

(ii) Random sample

(iii) Sampling distribution. 2½×4=10

4. (A) Define the chi-square statistic. State its p.d.f. Find mode of a Chi-square distribution. State and prove additive property of Chi-square distribution.

(B) Define Fisher's t. Derive its p.d.f. 5+5

OR

(E) Define F-statistic. Derive its p.d.f. Find μ'_r are hence find mean and variance of F-distribution. 10

(F) Given that H = {1, a^2} is a subgroup of group G = {a, a^2, a^3, a^4 = 1}. Then 10

5. Solve any **TEN** questions :

(A) Show that cov(aX, bY) = ab cov(X, Y).

(B) Find k if the joint p.d.f. of (X, Y) is

$$f(x, y) = \begin{cases} k(x + 2y) & , \quad 0 < x < 1,\ 0 < y < 1 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

(C) State the limits of correlation coefficient ρ_{xy}.

(D) If r.v. (X, Y) follows Bivariate normal distribution, state the conditional p.d.f. of Y given X = x.

(E) If r.v. (X, Y) follows Bivariate normal distribution with parameters $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho) \equiv (3, 2, 4, 9, 0.6)$ in usual notation, find the conditional mean of Y given X = 3.5.

(F) Write the p.d.f. of Bivariate normal distribution with parameters $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho) = (0, 0, 1, 1, \rho)$.

(G) If X ~ N(5, 1) then state the probability distribution of $(X - 5)^2$.

(H) If $X \sim N(\mu, \sigma^2)$, then state the probability distribution of $Y = a + bX$.

(I) Let X have a p.m.f.

$$f(x) = \begin{cases} \frac{1}{4} & , \quad x = 1, 2, 3, 4 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Find the p.m.f. of $Y = 2X$.

(J) If the m.g.f. of the distribution of r.v.x is $M_x(t) = (1 - 2t)^{-5/2}$, name the probability distribution of X and its mean.

(K) If $X_i \sim B(n_i, p)$, $i = 1, 2 \ldots\ldots n$ X_i are independent r.v.s. Then state the probability distribution of $\sum_{i=1}^{n} X_i$ with parameters.

(L) State mean of t-distribution and comment on its skewness. 1×10=10